Charged particles' tunneling from a noncommutative charged black hole

S. Hamid Mehdipour[1,*]

[1]*Islamic Azad University, Lahijan Branch, P. O. Box 1616, Lahijan, Iran*

(Dated: November 30, 2010)

Abstract

We apply the tunneling process of charged massive particles through the quantum horizon of a Reissner-Nordström black hole in a new noncommutative gravity scenario. In this model, the tunneling amplitude on account of noncommutativity influences in the context of coordinate coherent states is modified. Our calculation points out that the emission rate satisfies the first law of black hole thermodynamics and is consistent with an underlying unitary theory.

---

*Electronic address: mehdipour@iau-lahijan.ac.ir

I.   INTRODUCTION

Over three decades ago, Stephan Hawking [1] found that, utilizing the procedure of quantum field theory in curved spacetime, the radiation spectrum is almost like that of a black body, and can be described by a characteristic Hawking temperature with a purely thermal spectrum which yields to non-unitarity of quantum theory where maps a pure state to a mixed state. It has been proposed that Hawking radiation can be extracted from the null geodesic method suggested by Parikh and Wilczek [2]. In their method, they take the back-reaction effects into consideration and present a leading correction to the probability of massless particles tunneling across the horizon. The tunneling process clarifies that the extended radiation spectrum is not precisely thermal which leads to unitarity. Recently, Nicolini, Smailagic and Spallucci (NSS) [3] derived that black hole in a new model of noncommutativity does not allow to decay lower than a *minimal mass M_0*, i.e. black hole remnant (see also [4, 5]) . If we really believe the idea of stable black hole remnants due to the fact that there are some exact continuous global symmetries in nature [6], and also do not find any correlations between the tunneling rates of different modes in the black hole radiation spectrum, then these leave only one possibility: the information stays inside the black hole and can be retained by a stable Planck-sized remnant [7, 8] [58]. Although, this issue is then accepted if information conservation is really conserved in our universe. Thus we proceed our work with hope that this model of noncommutativity can provide a way to explain how the charged black hole decays, particularly in its final stages.

The NSS model of noncommutativity of coordinates that is carried on by the Gaussian distribution of coherent states, is consistent with Lorentz invariance, Unitarity and UV-finiteness of quantum field theory [10–14]. Since the noncommutativity of spacetime is an innate property of the manifold by itself even in absence of gravity, then some kind of divergences which emerge in general relativity and black hole physics, can be removed by it. Then with hope to cure the divergences of evaporation process of black hole physics we apply both back-reaction and noncommutativity effects to proceed the radiative process. The plan of this paper is the following. In Sec. II, we perform a brief discussion about the existence of black hole remnant within the noncommutative coordinate fluctuations at short distances (noncommutative inspired Reissner-Nordström solutions). We pay special attention to study of Hawking temperature. In Sec. III, a detailed calculation of quantum tunneling near the

smeared quantum horizon by considering a new model of noncommutativity is provided. The tunneling amplitude at which charged massive particles tunnel across the event horizon is computed and its applicability for the Reissner-Nordström black hole is discussed. And finally the paper is ended with summary (Sec. IV).

II. NONCOMMUTATIVE REISSNER-NORDSTRÖM BLACK HOLE

There exist many formulations of noncommutative field theory based on the Weyl-Wigner-Moyal $*$-product [15–17] that lead to failure in resolving of some important problems, such as Lorentz invariance breaking, loss of unitarity and UV divergences of quantum field theory. But recently, Smailagic and Spallucci [10–14] explained a fascinating model of noncommutativity, the coordinate coherent states approach, that can be free from the problems mentioned above. In this approach, a point-like mass M, and a point-charge Q instead of being quite localized at a point, are described by a smeared structure throughout a region of linear size $\sqrt{\theta}$. The approach we adopt here is to look for a static, asymptotically flat, spherically symmetric, minimal width, Gaussian distribution of mass and charge whose noncommutative size is determined by the parameter $\sqrt{\theta}$. To do this end, we shall model the mass and charge distributions by a smeared delta function ρ (see [3–5, 7, 8, 18–21])

$$
\begin{cases}
\rho_{matt.}(r) = \frac{M}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}} \\
\rho_{el.}(r) = \frac{Q}{(4\pi\theta)^{\frac{3}{2}}} e^{-\frac{r^2}{4\theta}}.
\end{cases}
\tag{1}
$$

The line element which solves Einstein's equations in the presence of smeared mass and charge sources can be obtained as

$$
ds^2 = -\left(1 - \frac{2M_\theta}{r} + \frac{Q_\theta^2}{r^2}\right)dt^2 + \left(1 - \frac{2M_\theta}{r} + \frac{Q_\theta^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2,
\tag{2}
$$

where M_θ and Q_θ are the smeared mass and charge distributions respectively and can implicity be given in terms of the lower incomplete Gamma function,

$$
\begin{cases}
M_\theta = \frac{2M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r^2}{4\theta}\right) \\
Q_\theta = \frac{Q}{\sqrt{\pi}}\sqrt{\gamma^2\left(\frac{1}{2}, \frac{r^2}{4\theta}\right) - \frac{r}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r^2}{2\theta}\right)} \\
\gamma\left(\frac{a}{b}, x\right) \equiv \int_0^x \frac{du}{u} u^{\frac{a}{b}} e^{-u}.
\end{cases}
$$

Throughout the paper, natural units are used with the following definitions; $\hbar = c = G = k_B = 1$. In the limit $\theta \to 0$, the noncommutative (modified) Reissner-Nordström solution

reduced to the commutative (ordinary) case and $M_\theta \to M$, $Q_\theta \to Q$ as expected. The outer and inner horizons of this line element can be found where $g_{00}(r_{\theta\pm}) = 0$, which are respectively given by

$$r_{\theta\pm}(r_{\theta\pm}) = M_\theta(r_{\theta\pm}) \pm \sqrt{M_\theta^2(r_{\theta\pm}) - Q_\theta^2(r_{\theta\pm})}. \tag{3}$$

The noncommutative horizon radius versus the mass and charge can approximately be calculated by setting $r_\pm = M \pm \sqrt{M^2 - Q^2}$ into the lower incomplete Gamma function as

$$r_{\theta\pm} \equiv r_{\theta\pm}(M,Q) = M_{\theta\pm} \pm \sqrt{M_{\theta\pm}^2 - Q_{\theta\pm}^2}, \tag{4}$$

with

$$\begin{cases} M_{\theta\pm} \equiv M_{\theta\pm}(M,Q) = M \left[\mathcal{E}\left(\frac{M\pm\sqrt{M^2-Q^2}}{2\sqrt{\theta}} \right) - \frac{M\pm\sqrt{M^2-Q^2}}{\sqrt{\pi\theta}} \exp\left(-\frac{\left(M\pm\sqrt{M^2-Q^2}\right)^2}{4\theta} \right) \right] \\ Q_{\theta\pm} \equiv Q_{\theta\pm}(M,Q) = Q\sqrt{\mathcal{E}^2\left(\frac{M\pm\sqrt{M^2-Q^2}}{2\sqrt{\theta}} \right) - \frac{M\pm\sqrt{M^2-Q^2}}{\sqrt{2\pi\theta}} \mathcal{E}\left(\frac{M\pm\sqrt{M^2-Q^2}}{\sqrt{2\theta}} \right)}, \end{cases}$$

where $\mathcal{E}(x)$ shows the *Gauss Error Function* defined as

$$\mathcal{E}(x) \equiv \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

For very large masses, the $\mathcal{E}\left(\frac{M\pm\sqrt{M^2-Q^2}}{2\sqrt{\theta}} \right)$ tends to unity and the exponential term will be reduced to zero and one retrieves the classical Reissner-Nordström horizons, $r_{\theta\pm} \to r_\pm = M \pm \sqrt{M^2 - Q^2}$.

The radiating behavior of such modified Reissner-Nordström black hole can now be found to calculate its temperature as follows

$$T_H = \frac{1}{4\pi} \frac{dg_{00}}{dr} \bigg|_{r=r_{\theta+}} = \frac{1}{4\pi r_{\theta+}} \left[1 - \frac{r_{\theta+}^3 \exp\left(-\frac{r_{\theta+}^2}{4\theta} \right)}{4\theta^{\frac{3}{2}}\gamma\left(\frac{3}{2}, \frac{r_{\theta+}^2}{4\theta} \right)} \right]$$

$$- \frac{Q^2}{\pi^2 r_{\theta+}^3} \left[\gamma^2\left(\frac{3}{2}, \frac{r_{\theta+}^2}{4\theta} \right) + \frac{r_{\theta+}^3 \exp\left(-\frac{r_{\theta+}^2}{4\theta} \right) \left(\gamma^2\left(\frac{1}{2}, \frac{r_{\theta+}^2}{4\theta} \right) - \frac{r_{\theta+}}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r_{\theta+}^2}{2\theta} \right) \right)}{16\,\theta^{\frac{3}{2}}\gamma\left(\frac{3}{2}, \frac{r_{\theta+}^2}{4\theta} \right)} \right]. \tag{5}$$

For the commutative case, $\left(\frac{M+\sqrt{M^2-Q^2}}{2\sqrt{\theta}} \right) \to \infty$, one recovers the classical Hawking temperature, $T_H = \frac{\sqrt{M^2-Q^2}}{2\pi\left(M+\sqrt{M^2-Q^2}\right)^2}$. The numerical calculation of such noncommutative Hawking temperature as a function of $r_{\theta+}$, for some values of Q is presented in Fig. 1. In this modi-
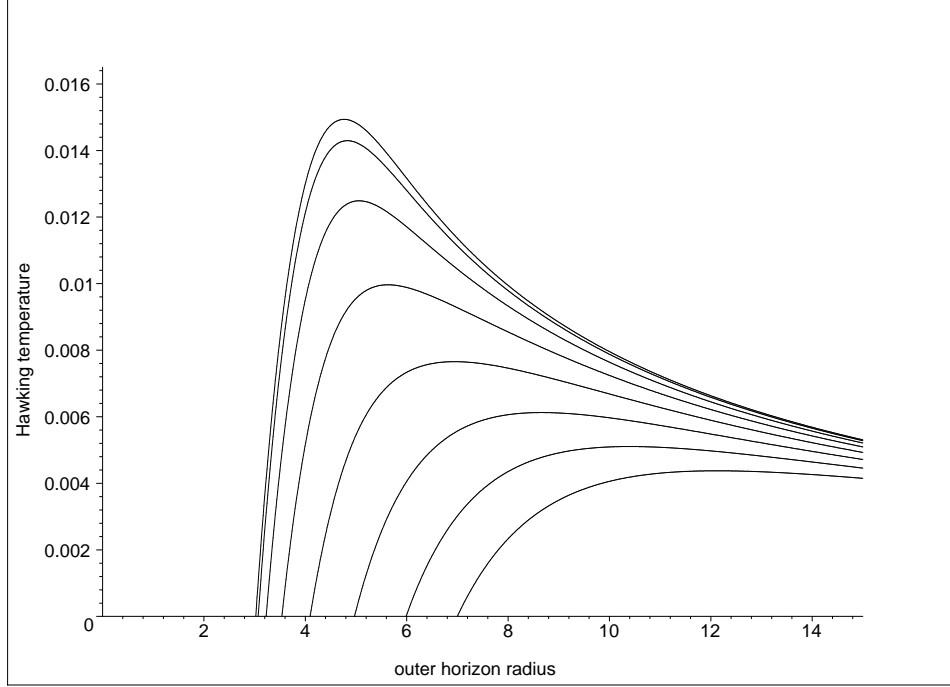
4

FIG. 1: Black hole temperature, $T_H \sqrt{\theta}$, as a function of $\frac{r_{\theta+}}{\sqrt{\theta}}$ (the outer horizon radius), for some values of Q. On the right-hand side of the figure, from top to bottom, the curves correspond to $Q = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6$ and 7, respectively. The top temperature reduces with growing Q. The existence of a minimal non-zero mass and disappearance of divergence are clear. In fact as a result of coordinate noncommutativity the black hole temperature falls to zero at the remnant mass.

fied version, not only the Hawking temperature does not diverge at all but also it reaches a maximum value before dropping to absolute zero at a minimal non-zero mass, $M = M_0$, that black hole shrinks to. In other words, Fig. 1 shows that coordinate noncommutativity yields to the existence of a minimal non-zero mass which black hole can reduce to. Therefore, in the noncommutative framework, black hole doesn't evaporate completely and this leads to a Planck-sized remnant including *the information*. So information might be preserved in this remnant. However, it is not conceivable to date to give a clear answer to the question of the black hole information paradox and this is reasonable because there is no complete self-consistent quantum theory of evaporating black holes (for reviews on resolving the so-called *information loss paradox*, see [22–27]).

In this situation, we should note that our calculation to obtain the Eq. (5) is accurate and no approximation has been made. If we want to acquire the simple $r_{\theta\pm}$-dependent form of the noncommutative Hawking temperature, then it can be approximated as follows

$$T_H = \frac{\kappa(M,Q)}{2\pi} \approx \frac{1}{4\pi} \frac{r_{\theta+} - r_{\theta-}}{r_{\theta+}^2}, \tag{6}$$

where $\kappa(M, Q)$ is the noncommutative surface gravity at the horizon and is given by

$$\kappa(M, Q) \approx \frac{r_{\theta+} - r_{\theta-}}{2r_{\theta+}^2}. \tag{7}$$

In Sec. III, we will use this approximate expression to compute the tunneling probability when the first law of black hole thermodynamics is applied (see Eq. (25)).

III.   PARIKH-WILCZEK TUNNELING AS CHARGED MASSIVE PARTICLES

We are now ready to discuss the quantum tunneling process in the noncommutative framework. To describe this procedure, where a particle moves in dynamical geometry and pass through the horizon without singularity on the path we should use the coordinates systems that, unlike Reissner-Nordström coordinates, are not singular at the horizon (the outer horizon). A particularly convenient choice is Painlevé coordinate [28] which is obtained by definition of a new noncommutative time coordinate,

$$dt = dt_r + \frac{r\sqrt{2M_{\theta+}r - Q_{\theta+}^2}}{r^2 - 2M_{\theta+}r + Q_{\theta+}^2} dr = dt_r + dt_{syn}, \tag{8}$$

where t_r is the Reissner time coordinate, and

$$dt_{syn} = -\frac{g_{01}}{g_{00}} dr. \tag{9}$$

Note that only the Reissner time coordinate is transformed. Both the radial coordinate and angular coordinates remain the same. The noncommutative Painlevé metric now immediately reads

$$ds^2 = g_{00}dt^2 + 2g_{01}dtdr + g_{11}dr^2 + g_{22}d\vartheta^2 + g_{33}d\varphi^2$$

$$= -\left(1 - \frac{2M_{\theta+}}{r} + \frac{Q_{\theta+}^2}{r^2}\right)dt^2 + 2\sqrt{\frac{2M_{\theta+}}{r} - \frac{Q_{\theta+}^2}{r^2}}dtdr + dr^2 + r^2(d\vartheta^2 + \sin^2\vartheta d\varphi^2). \tag{10}$$

It should be stressed here that the Eq. (9), in accord with Landau's theory of the synchronization of clocks [29], allows us to synchronize clocks in any infinitesimal radial positions of space $(d\vartheta = d\varphi = 0)$. Since the tunneling phenomena through the quantum horizon i.e. *the barrier* is an instantaneous procedure it is important to consider Landau's theory of the coordinate clock synchronization in the tunneling process. The mechanism for tunneling through the quantum horizon is that particle anti-particle pair is created at the event horizon. So, we have two events that occur simultaneously; one event is anti-particle and tunnels

into the barrier but the other particle tunnels out the barrier. In fact, the Eq. (9) mentions the difference of coordinate times for these two simultaneous events occurring at infinitely adjacent radial positions. Furthermore, the noncommutative Painlevé-Reissner-Nordström metric exhibits the stationary, non-static, and neither coordinate singularity nor intrinsic singularity.

To obtain the radial geodesics of the charged massive particles' tunneling across the potential barrier which are different with the uncharged massless [7] and also the uncharged massive [8] ones, we follow the same noncommutative method in this work but now extended to the case of charged massive particles. According to the non-relativistic quantum theory, de Broglie's hypothesis and the WKB approximation, it can be easily proved that the treatment of the massive particle's tunneling as a massive shell is approximately derived by the phase velocity v_p of the de Broglie s-wave whose the relationship between phase velocity v_p and group velocity v_g is given by [30–32]

$$v_p = \dot{r} = \frac{1}{2}v_g, \tag{11}$$

overdot abbreviates $\frac{d}{dt}$. In the case of $d\vartheta = d\varphi = 0$, according to the relation (9), the group velocity is

$$v_g = -\frac{g_{00}}{g_{01}}. \tag{12}$$

Thus, the outgoing motion of the massive particles take the form

$$\dot{r} = -\frac{g_{00}}{2g_{01}} = \frac{r^2 - 2M_{\theta+}r + Q_{\theta+}^2}{2r\sqrt{2M_{\theta+}r - Q_{\theta+}^2}}. \tag{13}$$

If we suppose that t increases towards the future then the above equations will be modified by the particle's self-gravitation effect [33, 34] (see also [35, 36]) . To assure the conservation of energy and electric charge, we fix the total ADM mass (M) and electric charge (Q) of the spacetime and permit the hole mass and its charge to fluctuate. In other words, we should replace M by $M - E$ and Q by $Q - q$ both in the Eqs. (10) and (13), because the response of the background geometry is taken into account by an emitted quantum of energy E with electric charge q. Thus, when a charged particle tunnels out, the black holes's mass and also electric charge will change for the conservation of energy and charge.

In order to consider the effect of the electromagnetic field, it is necessary to take into account Maxwell gravity system comprises of the black hole and the electromagnetic field

outside the hole. The lagrangian function of the Maxwell gravity system should be written as

$$L = L_{matt.} + L_{el.},\tag{14}$$

where $L_{el.} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ is the lagrangian function, while the Maxwell field $F^{\mu\nu}$ must take on the form

$$F^{\mu\nu} = \delta^{0[\mu}\delta^{r]\nu]} E(r\,;\,\theta) = \partial^\mu\phi^\nu - \partial^\nu\phi^\mu.\tag{15}$$

Studying the behavior of Coulomb-like field within the noncommutativity framework have already been investigated in Refs. [4, 5]. The Electric field $E(r\,;\,\theta)$ is found by solving the following Maxwell equations with a Gaussian-profile of smearing-charge source along the time direction:

$$\frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,F^{\mu\nu}\right) = \rho_{el.}\,\delta_0^\nu,\tag{16}$$

which is given as

$$E(r\,;\,\theta) = \frac{2Q}{\sqrt{\pi}\,r^2}\,\gamma\left(\frac{3}{2},\frac{r^2}{4\theta}\right).\tag{17}$$

The regular behavior of the Coulomb-like field at the origin is clear. In the limit of $\theta \to 0$, the lower incomplete Gamma function reduces to the complete Gamma function $\Gamma(\frac{3}{2})$, and one recovers the ordinary Coulomb field. The last equality of the Eq. (15) defines $F^{\mu\nu}$ in terms of the 4-potential and corresponds to the generalized coordinates $\phi_\mu = (\phi, 0, 0, 0)$ [37], where the Coulomb-like potential ϕ is the only non-zero component of the electromagnetic potential ϕ_μ and can be obtained as follows:

$$\phi \equiv \phi(r\,;\,\theta) =$$

$$-\frac{Q}{2\sqrt{\pi\theta}}\left[\frac{r^2}{12\theta}\,{}_3F_3\left(1,1,\frac{3}{2}\,;\,2,2,\frac{5}{2}\,;\,-\frac{r^2}{4\theta}\right) - \Gamma\left(0,\frac{r^2}{4\theta}\right) - 2\ln\left(\frac{r^2}{\theta}\right) + 2\ln 2 + 2 - \gamma\right].\tag{18}$$

In the above equation, ${}_pF_q$ shows the *hypergeometric series* defined in terms of the *pochhammer symbol* as follows:

$${}_pF_q\left(a_1,\ldots,a_p\,;\,b_1,\ldots,b_q\,;\,z\right) = \sum_{n=0}^\infty \frac{z^n\left(\prod_{i=1}^p pochhammer(a_i,n)\right)}{n!\left(\prod_{j=1}^q pochhammer(b_j,n)\right)}.$$

where, for example, $pochhammer(a_i, n) = a_i(a_i + 1)\ldots(a_i + n - 1)$. The second term, $\Gamma\left(0, \frac{r^2}{4\theta}\right)$, is the upper incomplete Gamma function which is defined as

$$\Gamma\left(0, \frac{r^2}{4\theta}\right) = \int_{\frac{r^2}{4\theta}}^\infty \frac{e^{-t}}{t}dt.$$

8

The last term of the Eq. (18), γ, is the *Euler's constant* that is approximately equal to 0.577215. To have a clear depiction, we have computed the numerical results of both the noncommutative Coulomb-like potential and commutative case which are shown in Fig. 2. When a charged particle passes through the event horizon, the whole system will transit
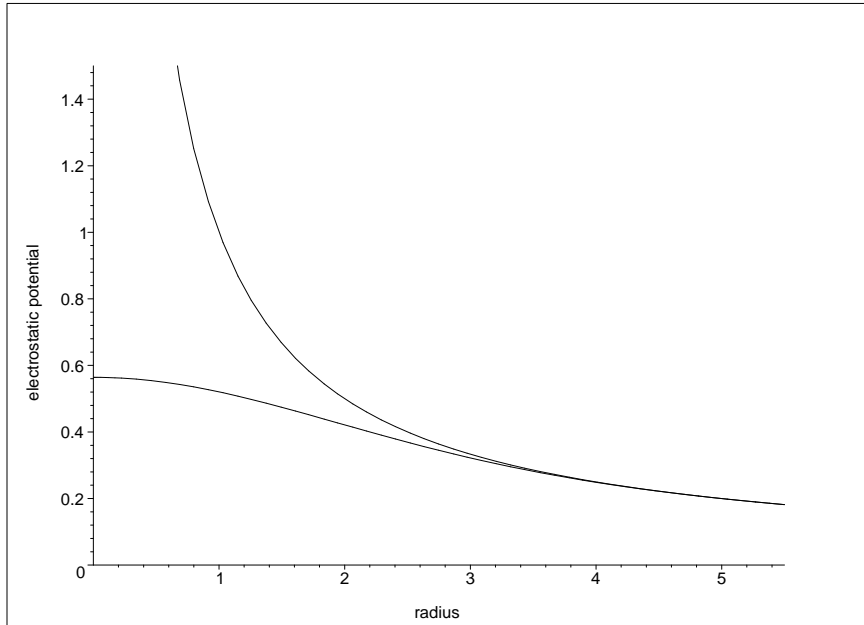

FIG. 2: The electrostatic potential (per unit charge) versus the radius. The upper curve is the ordinary Coulomb potential and the lower curve is the result of noncommutativity. The regular behavior of the the Coulomb-like potential at the origin can be easily seen from the lower curve.

from one state to another. In order to remove the freedom equivalent to ϕ, due to the fact that it is an ignorable coordinate, the action is then found to be

$$I = \int_{t_{in}}^{t_{out}} \left(L - P_\phi \dot{\phi} \right) dt, \tag{19}$$

where P_ϕ is the canonical momentum conjugate to ϕ. Since the characteristic wavelength of the radiation is always haphazardly small near the horizon due to the infinite blue-shift there, so that the wave-number reaches infinity and the WKB approximation is reliable close to the horizon. In the WKB approximation, the probability of tunneling or emission rate for the classically forbidden region as a function of the imaginary part of the particle's action at stationary phase would take the form [59]

$$\Gamma \sim \exp(-2\mathrm{Im}\,I). \tag{20}$$

To calculate the imaginary part of the action we consider a spherical shell to consist of components of the charged massive particles each of which travels on a radial timelike

9

geodesic, so that we will use these radial timelike geodesics like an s-wave outgoing positive energy particle which pass through the horizon outwards from r_{in} to r_{out} to compute the $\text{Im}\,I$, as follows

$$\text{Im}\,I = \text{Im} \int_{r_{in}}^{r_{out}} \left(P_r - \frac{P_\phi \dot{\phi}}{\dot{r}} \right) dr = \text{Im} \int_{r_{in}}^{r_{out}} \left[\int_{(0,0)}^{(P_r,P_\phi)} dP_r' - \frac{\dot{\phi}}{\dot{r}} dP_\phi' \right] dr, \qquad (21)$$

we now alter the integral variables by using Hamilton's equations

$$\begin{cases} \dot{r} = \frac{dH}{dP_r}\big|_{(r;\,\phi,\,P_\phi)} = \frac{d(M-E)}{dP_r} = -\frac{dE}{dP_r} \\ \dot{\phi} = \frac{dH}{dP_\phi}\big|_{(\phi;\,r,\,P_r)} = \phi(Q-q)\,\frac{dq}{dP_\phi}. \end{cases} \qquad (22)$$

Hence, the imaginary part of the action gives the following expression:

$$\text{Im}\,I = -\text{Im} \int_{r_{in}}^{r_{out}} \left[\int_{(0,0)}^{(E,q)} \frac{dE'}{\dot{r}} + \frac{\phi(Q-q')}{\dot{r}} dq' \right] dr. \qquad (23)$$

Substituting the expression of $\dot{r}$ from (13) into (23), under the condition that the self-gravitation effect of the particle itself is included, we have,

$$\text{Im}\,I = -\text{Im} \int_{r_{in}}^{r_{out}} \left[\int_{(0,0)}^{(E,q)} \frac{2r\sqrt{2M'_{\theta+}r - Q'^2_{\theta+}}}{r^2 - 2M'_{\theta+}r + Q'^2_{\theta+}} dE' + \frac{2r\,\phi(Q-q')\sqrt{2M'_{\theta+}r - Q'^2_{\theta+}}}{r^2 - 2M'_{\theta+}r + Q'^2_{\theta+}} dq' \right] dr, \qquad (24)$$

where

$$\begin{cases} M'_{\theta+} \equiv M_{\theta+}(M - E', Q - q') \\ Q'_{\theta+} \equiv Q_{\theta+}(M - E', Q - q'). \end{cases}$$

The r integral has a pole at the outer horizon where lies along the line of integration. This integral can be done first by deforming the contour and it yields to $(-\pi i)$ times the residue. Note that we require $r_{in} > r_{out}$ where:

$$\begin{cases} r_{in} = \frac{2M}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r_{in}^2}{4\theta}\right) + \sqrt{\frac{4M^2}{\pi}\gamma^2\left(\frac{3}{2}, \frac{r_{in}^2}{4\theta}\right) - \frac{Q^2}{\pi}\left(\gamma^2\left(\frac{1}{2}, \frac{r_{in}^2}{4\theta}\right) - \frac{r_{in}}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r_{in}^2}{2\theta}\right)\right)} \\ r_{out} = \frac{2(M-E)}{\sqrt{\pi}}\gamma\left(\frac{3}{2}, \frac{r_{out}^2}{4\theta}\right) + \sqrt{\frac{4(M-E)^2}{\pi}\gamma^2\left(\frac{3}{2}, \frac{r_{out}^2}{4\theta}\right) - \frac{(Q-q)^2}{\pi}\left(\gamma^2\left(\frac{1}{2}, \frac{r_{out}^2}{4\theta}\right) - \frac{r_{out}}{\sqrt{2\theta}}\gamma\left(\frac{1}{2}, \frac{r_{out}^2}{2\theta}\right)\right)} \end{cases}$$

Thus,

$$\text{Im}\,I = 2\pi \left[\int_{(0,0)}^{(E,q)} \frac{r'^2_{\theta+}}{r'_{\theta+} - r'_{\theta-}} dE' + \frac{r'^2_{\theta+}\,\phi(Q-q')}{r'_{\theta+} - r'_{\theta-}} dq' \right] = \pi \left[\int_{(0,0)}^{(E,q)} \frac{dE'}{\kappa'} - \frac{V'}{\kappa'} dq' \right], \qquad (25)$$

where

$$\begin{cases} r'_{\theta\pm} \equiv r_{\theta\pm}(M - E', Q - q') \\ V' = -\phi(Q - q')\big|_{r=r'_{\theta+}} \\ \kappa' \equiv \kappa(M - E', Q - q') = \frac{r'_{\theta+} - r'_{\theta-}}{2r'^2_{\theta+}}. \end{cases}$$

10

In the above expressions, V' and κ', respectively, are the electro-potential on the event horizon and the horizon surface gravity in which self-gravitations are comprised. Here, utilizing the first low of black hole thermodynamics, $dM = \frac{\kappa}{2\pi}dS + VdQ$, one can find the imaginary part of the action as [44–53]

$$\text{Im}\, I = -\frac{1}{2}\int_{S_{NC}(M,Q)}^{S_{NC}(M-E,Q-q)} dS = -\frac{1}{2}\Delta S_{NC}(M,Q,E), \qquad (26)$$

where $\Delta S_{NC}(M,Q,E)$ is the difference in noncommutative black hole entropies before and after emission. Since, both particle and anti-particle (which corresponds to a time reversed situation and it can be seen that as backward in time by replacing $\sqrt{\frac{2M_{\theta+}}{r} - \frac{Q_{\theta+}^2}{r^2}}$, by $-\sqrt{\frac{2M_{\theta+}}{r} - \frac{Q_{\theta+}^2}{r^2}}$, in the metric (10)) anticipate in the emission rate for the Hawking process via tunneling with same amounts, therefore we should have to add their amplitudes first and then to square it to obtain the emission probability,

$$\Gamma \sim \exp(-2\text{Im}\, I) \sim \exp\left(\Delta S_{NC}(M,Q,E)\right) = \exp[S_{NC}(M-E,Q-q) - S_{NC}(M,Q)]. \quad (27)$$

Hawking radiation as tunneling was also investigated in the context of black holes in string theory [44], and it was exhibited that the emission rates in the high energy corresponds to a difference between counting of states in the microcanonical and canonical ensembles. Thus at higher energies the emission spectrum cannot be precisely thermal due to the fact that the high energy corrections arise from the physics of energy and charge conservation with noncommutativity corrections. In fact, the emission rate (27) deviates from the pure thermal emission but is consistent with an underlying unitary quantum theory and support the conservation of information [54]. The question which arises here is the possible dependencies between different modes of radiation during the evaporation [55–57] and then the time-evolution of these possible correlations which needs further investigation and probably shed more light on information loss problem. This problem is currently under investigation.

IV. SUMMARY

The generalization of the standard Hawking radiation via tunneling through the event horizon based on the solution of the Eq. (20) within the context of coordinate coherent state noncommutativity has been studied and then the new corrections of the emission rate based on noncommutative framework has been achieved. To describe the noncommutative

behavior of an electro-gravitational system, we have extended the Parikh-Wilczek tunnelling process to calculate the tunneling probability of a charged massive particle from the Reissner-Nordström black hole within the framework of noncommutative quasi coordinates. Studying its behavior shows that, as expected, the emission rate is consistent with the unitary theory and satisfies the first law of black hole thermodynamics.

[1] S. W. Hawking, Comm. Math. Phys. **43**, 199 (1974).

[2] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000).

[3] P. Nicolini, A. Smailagic and E. Spallucci, Phys. Lett. B **632**, 547 (2006).

[4] S. Ansoldi, P. Nicolini, A. Smailagic and E. Spallucci, Phys. Lett. B **645**, 261 (2007).

[5] P. Nicolini, Int. J. Mod. Phys. A **24**, 1229 (2009).

[6] J. D. Bekenstein, Phys. Rev. D **5**, 1239 (1972).

[7] K. Nozari and S. H. Mehdipour, Class. Quant. Grav. **25**, 175015 (2008).

[8] S. H. Mehdipour, Commun. Theor. Phys. **52**, 865 (2009).

[9] S. H. Mehdipour, Phys. Rev. D **81**, 124049 (2010).

[10] A. Smailagic and E. Spallucci, Phys. Rev. D **65**, 107701 (2002).

[11] A. Smailagic and E. Spallucci, J. Phys. A **35**, L363 (2002).

[12] A. Smailagic and E. Spallucci, J. Phys. A **36**, L467 (2003).

[13] A. Smailagic and E. Spallucci, J. Phys. A **36**, L517 (2003).

[14] A. Smailagic and E. Spallucci, J. Phys. A **37**, 7169 (2004).

[15] H. Weyl, Z. Phys. **46**, 1 (1927).

[16] E. Wigner, Phys. Rev. **40**, 749 (1932).

[17] J. E. Moyal, Proc. Camb. Phil. Soc. **45**, 99 (1949).

[18] T. G. Rizzo, JHEP **0609**, 021 (2006).

[19] R. Casadio and P. Nicolini, JHEP **0811**, 072 (2008).

[20] E. Spallucci, A. Smailagic and P. Nicolini, Phys. Lett. B **670**, 449 (2009).

[21] K. Nozari and S. H. Mehdipour, Commun. Theor. Phys. **53**, 503 (2010).

[22] J. Preskill, [arXiv:hep-th/9209058].

[23] D. N. Page, Phys. Rev. Lett. **71**, 3743 (1993).

[24] C. R. Stephens, G. 't Hooft and B. F. Whiting, Class. Quant. Grav. **11**, 621 (1994).

[25] A. Strominger, [arXiv:hep-th/9501071].

[26] T. Banks, Nucl. Phys. (Proc. Suppl.) **41**, 21 (1995).

[27] J. G. Russo, [arXiv:hep-th/0501132] and references therein.

[28] P. Painlevé, Compt. Rend. Acad. Sci. (Paris) **173**, 677 (1921).

[29] L. D. Landau and E. M. Lifshitz, *The classical theory of field*, Pergamon Press, London, p.254 (1975).

[30] J. Zhang and Z. Zhao, Nucl. Phys. B **725**, 173 (2005).

[31] J. Zhang and Z. Zhao, JHEP **0510**, 055 (2005).

[32] Q. Q. Jiang, S. Q. Wu and X. Cai, Phys. Rev. D **73**, 064003 (2006).

[33] P. Kraus and F. Wilczek, Nucl. Phys. B **433**, 403 (1995).

[34] P. Kraus and F. Wilczek, Nucl. Phys. B **437**, 231 (1995).

[35] S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, Class. and Quant. Grav. **19**, 2671 (2002).

[36] S. Shankaranarayanan, T. Padmanabhan and K. Srinivasan, Mod. Phys. Lett. A **16**, 571 (2001).

[37] J. Mäkela and P. Repo, Phys. Rev. D **57**, 4899 (1998).

[38] E. T. Akhmedov, V. Akhmedova and D. Singleton, Phys. Lett. B **642**, 124 (2006).

[39] E. T. Akhmedov, V. Akhmedova, T. Pilling and D. Singleton, Int. J. Mod. Phys. A **22**, 1705 (2007).

[40] B. D. Chowdhury, Pramana **70**, 593 (2008).

[41] T. Pilling, Phys. Lett. B **660**, 402 (2008).

[42] V. Akhmedova, T. Pilling, A. de Gill and D. Singleton, Phys. Lett. B **666**, 269 (2008).

[43] E. T. Akhmedov, T. Pilling and D. Singleton, Int. J. Mod. Phys. D **17**, 2453 (2008).

[44] E. Keski-Vakkuri and P. Kraus, Nucl. Phys. B **491**, 249 (1997).

[45] S. Massar and R. Parentani, Nucl. Phys. B **575**, 333 (2000).

[46] A. J. M. Medved and E. Vagenas, Mod. Phys. Lett. A **20**, 1723 (2005).

[47] A. J. M. Medved and E. Vagenas, Mod. Phys. Lett. A **20**, 2449 (2005).

[48] M. Arzano, A. Medved and E. Vagenas, JHEP **0509**, 037 (2005).

[49] R. Banerjee, B. R. Majhi and S. Samanta, Phys. Rev. D **77**, 124035 (2008).

[50] R. Banerjee and B. R. Majhi, Phys. Lett. B **662**, 62 (2008).

[51] R. Banerjee and B. R. Majhi, JHEP **06**, 095 (2008).

[52] R. Banerjee, B. Chakraborty, S. Ghosh, P. Mukherjee and S. Samanta, Found. Phys. **39**, 1297 (2009).

[53] R. Banerjee, B. R. Majhi and S. K. Modak, Class. Quant. Grav. **26**, 085010 (2009).

[54] M. K. Parikh, Int. J. Mod. Phys. D **13**, 2351 (2004).

[55] K. Nozari and S. H. Mehdipour, Europhys. Lett. **84**, 20008 (2008).

[56] S. H. Mehdipour, Int. J. Mod. Phys. A **24**, 5669 (2009).

[57] K. Nozari and S. H. Mehdipour, JHEP **03**, 061 (2009).

[58] However, in Ref. [9], we have analyzed a specific form of the noncommutative-inspired Vaidya solution which leads to a zero remnant mass in the long-time limit, i.e. an instable black hole remnant.

[59] We should stress that there is another opinion on utilizing the expression (20). There is a problem here recognized as "factor 2 problem" [38, 39]. Lately, some authors (see [40, 41] and references therein) have stated that the expression (20) is not invariant under canonical transformations however the same formula with a factor of 1/2 in the exponent is canonically invariant. This procedure yields a temperature higher than the Hawking temperature by a factor of 2. In Refs. [42, 43], a resolution to this problem was given in terms of an overlooked temporal contribution to the tunneling amplitude. When one includes this temporal contribution one gets exact the correct temperature and exactly when one uses the canonically invariant tunneling amplitude.